# The formulation of prime numbers 

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#### Abstract

Prime numbers are essential in various mathematical disciplines, yet their identification continues to challenge researchers. In this brief report, we introduce a unique solution to formulate prime numbers. Using a process of elimination based on a list of the natural numbers as six functions, we systematically eliminate all composite candidates to arrive at the final prime result. In addition, we explore how our model could enhance the efficiency of the Sieve of Eratosthenes in identifying prime numbers.


In this paper, we will attempt to discover exact solutions for prime numbers as function formulas. Let us make a list of the natural numbers as six functions of $n$, where $n$ is a natural number.

| $\boldsymbol{n}$ | $\mathbf{2 ( 3 n - 2 )}$ | $\mathbf{3 ( 2 n - 1 )}$ | $\mathbf{2 ( 3 n - 1 )}$ | $\mathbf{6 n - 1}$ | $\mathbf{2} \mathbf{3} \boldsymbol{n}$ | $\mathbf{6} \boldsymbol{n}+\mathbf{1}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | 2 | 3 | 4 | 5 | 6 | 7 |
| $\mathbf{2}$ | 8 | 9 | 10 | 11 | 12 | 13 |
| $\mathbf{3}$ | 14 | 15 | 16 | 17 | 18 | 19 |
| $\mathbf{4}$ | 20 | 21 | 22 | 23 | 24 | $\mathbf{2}$ |
| $\mathbf{5}$ | 26 | 27 | 28 | 29 | 30 | 31 |
| $\mathbf{6}$ | 32 | 33 | 34 | 35 | 36 | 37 |
| $\mathbf{7}$ | 38 | 39 | 40 | 41 | 42 | 43 |
|  | $\ldots$ |  |  |  |  |  |

We state the following three observations about this list.

1) The second, third, fourth, and sixth columns contain the prime factor(s) 2 and/or 3.
2) As a result of observation 1), all prime numbers except 2 and 3 are in the fifth and seventh columns. Hence, all primes except 2 and 3 can be represented as $6 n-1$ or $6 n+1$.
3) Because all numbers in the fifth and seventh columns never contain the prime factor 2 or 3, all composite numbers in those columns can be expressed as ( $6 a-1$ )( $6 b-1$ ) or $(6 a-1)(6 b+1)$ or $(6 a+1)(6 b+1)$, where $a$ and $b$ are natural numbers.

As a result, the prime numbers can be determined as follows:
prime numbers $=2,3,6 n-1,6 n+1$ for $n=1,2, \ldots$, except when $6 n-1$ or $6 n+1=(6 a-1)(6 b-1)$ or $(6 a-1)(6 b+1)$ or $(6 a+1)(6 b+1)$ for $a, b=1,2, \ldots$.

## Discussion

Although the algorithm developed in this paper may use more computational resources than the well-known Sieve of Eratosthenes, it doesn't require knowledge of prime numbers during the computation process, whereas the Sieve of Eratosthenes does rely on such knowledge. Therefore, this algorithm enables us to identify prime numbers within a specific range of natural numbers. Moreover, by dividing these numbers into smaller blocks, it becomes simpler to perform parallel computations.

