Energy and Spacetime

Revised Edition:
Standard–Model Alignment and a Six–Dimensional Calabi–Yau Extension

Ryoji Furui
www.ryoji.info

Abstract

This paper proposes a modified framework for special relativity in which mass is carried by the temporal component of the Minkowski metric, so that a particle may convert between a massive (subluminal) and a massless (luminal) state. Because the standard relativistic energy relations already permit a particle to convert all of its rest energy into kinetic energy, the framework reaches the light–speed limit without the usual infinite–energy obstruction. The conversion produces gravitons, which transfer energy and momentum to matter by collision and thereby generate the gravitational force; the time symmetry of the underlying oscillation suggests an eternal universe.

The paper has two parts. **Part I** develops the original framework: the temporal mass term ζ (§2), the mass–energy oscillation (§3), graviton kinematics (§4), and a modified Einstein equation (§5). It then supplies a Lagrangian formulation with a symmetry analysis and a four–momentum derivation of the graviton relations (§6), and checks the model against general relativity and quantum field theory (§7). **Part II** aligns and extends the framework. The single scalar ζ is promoted to a family of energy–dependent functions by identifying it with the temporal element of the *pentadimensional* interaction metrics of Deformed Space–Time (DST) theory, in which energy is a measurable fifth coordinate (§9). The construction is then lifted to a six–dimensional Calabi–Yau internal manifold (§10), with the $T^6/\mathbb{Z}_3$ orbifold worked out explicitly—Hodge numbers, Euler characteristic, and a three–generation refinement (§10.7). A dictionary maps each Standard–Model particle to a geometric feature of the manifold (§11) and to the tetrahedral "Dice–o–topes" nuclear model (§12), and the model is confronted with observational constraints—sub–millimetre gravity, GW170817, collider missing energy, and the dark sector (§13).

Note on Authorship and Review Status

The following applies to the authorship and review status of this document.

- **Part I (The Original Framework)** mostly represents the author's original work.
- **Part II (Alignment and Extension)** was drafted by Anthropic's Claude (commercial tier) at the author's direction, incorporating reviewer comments from Microsoft Copilot during a subsequent revision pass.

The author confirms that they do not hold formal training in the relevant theories (special and general relativity, quantum field theory, and string compactification) and have not obtained independent expert review. Readers should therefore treat the construction and its physical and geometric claims as a starting point for discussion rather than as peer–reviewed physics. Formal expert review must be obtained before any part of this framework is relied upon as established science.

Contents

1 Introduction **3**

2 Mass term in the Minkowski metric **3**

3 Mass–energy wavefunction **4**

4 Properties of gravitons **4**

5 Gravitational field as universe **5**

6 Lagrangian formulation and symmetries **6**
6.1 The mass–energy oscillator as a Lagrangian system . . . 6
6.2 Action for the temporal mass term . . . 6
6.3 Symmetry analysis: Lorentz invariance is local, not global . . . 7
6.4 Graviton kinematics from four–momentum conservation . . . 7

7 Consistency with established physics **8**

8 Summary of the original framework **8**

9 Energy as a coordinate: aligning ζ with the Standard Model **9**
9.1 The pentadimensional energy metrics . . . 9
9.2 Two metric types are two interaction sectors . . . 9
9.3 The identification $\zeta^2 = 1/G$ . . . 10
9.4 The fifth element, the threshold energies, and a first appearance of the number six . . . 10

10 From five to six dimensions: a Calabi–Yau extension **11**
10.1 The isomap as a unified coordinate . . . 11
10.2 Kaluza–Klein reduction and the energy circle . . . 11
10.3 The Calabi–Yau threefold . . . 12
10.4 Topology: Hodge numbers, the Euler characteristic, and generations . . . 13
10.5 The exponent dictionary . . . 13
10.6 The honeycomb fibre and the hexagonal lattice . . . 14
10.7 A worked compactification: the $T^6/\mathbb{Z}_3$ orbifold . . . 14
10.8 The three–layer architecture . . . 15

11 Standard–Model particles as geometric shapes of the 6D manifold **16**

12 Bridge to the geometric nuclear model **17**

13 Observational constraints and experimental signatures **19**
13.1 The length scale: large extra dimensions versus sub–millimetre gravity . . . 19
13.2 Gravitational sector . . . 19
13.3 Collider sector: the half–mass signature . . . 19
13.4 Cosmological sector . . . 19

14 Discussion **20**

15 Conclusions **20**

Acknowledgments **21**

Part I. The Original Framework

1 Introduction

Special relativity (STR) is experimentally well confirmed but is usually regarded as silent about gravity and the nature of mass. The standard reason is that gravity involves acceleration and non–inertial frames, whereas STR is by construction a theory of inertial frames—though an accelerated frame can be built from a continuous sequence of inertial ones. General relativity (GTR) supplies gravity, but its purely geometric treatment sits awkwardly beside the way the other forces are described, and reconciling it with quantum theory remains open.

This paper asks whether STR itself, suitably modified, can accommodate mass and a quantized gravity with gravitons. The logic of Part I is as follows. Section 2 places mass inside the metric by introducing a mass term ζ in the time component, giving a modified Minkowski geometry. Section 3 shows that a massive particle can convert into a massless one travelling at c, and casts this as an oscillation that also supplies a graviton production mechanism. Section 4 treats gravitons within STR as carriers that impart energy and momentum to matter by collision, and reads off attractive and repulsive branches. Section 5 recasts the picture as a modified Einstein equation. Section 6 then puts the postulates of the earlier sections on a firmer footing: it gives a Lagrangian for the mass term and the oscillation, analyses what happens to Lorentz invariance, and derives the graviton relations from four–momentum conservation. Section 7 checks the model against established gravitational and quantum field theory. Part II (§9 onward) then aligns the framework with the Standard Model and extends it to a six–dimensional Calabi–Yau geometry.

2 Mass term in the Minkowski metric

In four–dimensional Minkowski spacetime the squared norm of the position vector $X(t,x,y,z)$ is $X^2 = t^2 - x^2 - y^2 - z^2$, with signature $(+,-,-,-)$ and $c = 1$. In an object's rest frame $X_0 = (t,0,0,0)$, so ${X_0}^2 = t^2$. Since time carries the object's energy and, at rest, all of that energy is mass–energy, it is natural to treat mass as intrinsic to the time dimension. We therefore introduce a mass–dependent term ζ into the time component of the metric, so that placing a mass in the rest frame contracts the four–vector:

$$ {X_0}^2 = \zeta^2 t^2, \qquad 0 \le \zeta < 1, \tag{1} $$

where ζ encodes the mass and $\zeta = 0$ is the state of maximum mass density.

Equation (1) appears to break Lorentz invariance, but the appearance is superficial. An observer who undergoes the transformation $\zeta = 1 \to \zeta = 0$ finds mass generated in an otherwise flat rest frame (and removed under the inverse operation). This is a different operation from a boost of a spatially moving frame, and it preserves Lorentz invariance provided the generated mass is embedded in flat spacetime—a statement made precise in §6.3. To unify gravity with the other forces, ζ is carried by the time–diagonal element of the relevant metric; the restricted range $0 \le \zeta < 1$ then allows a particle to oscillate between a pure–energy and a pure–rest–energy state, which is the subject of the next section.

3 Mass–energy wavefunction

A massive particle is usually said to be unable to reach c because doing so would require infinite energy. The relativistic energy relation tells a more permissive story: a particle can reach c by converting its rest energy into kinetic energy, without importing energy from outside. With $E^2 = m^2 + |\mathbf{p}|^2$, a particle at rest ($E = m$) can in principle convert its rest energy entirely into momentum and reach the massless state $E = |\mathbf{p}|$. Quantum mechanically we model this as a particle P oscillating between its massive and massless states in 1+1 dimensions. Taking P at $x = 0$ with $v = 1$ at $t = 0$, the oscillation is

$$x = \frac{T}{2\pi} \sin\left(2\pi \frac{t}{T}\right), \tag{2}$$

with period T, so that the velocity is

$$v = \cos\left(2\pi \frac{t}{T}\right). \tag{3}$$

Over $0 \le t < T/4$ and $T/2 \le t < 3T/4$ the kinetic energy converts fully into rest energy ($\zeta = 0$); over $T/4 \le t < T/2$ and $3T/4 \le t < T$ the rest energy converts fully into kinetic energy with $v = 1$. Because one period T contains the conversion of all of P's energy, and mass and energy are intrinsic to time, we take the relativistic energy to be linear in T, $E = AT$, with A constant. In terms of the oscillation frequency $f = 1/T$,

$$E = \frac{A}{f}. \tag{4}$$

A particle moving in more dimensions, whether massive or massless, decomposes into a collection of such 1+1 oscillators. For the photon branch, identifying $1/f = \nu$ with the photon frequency and $E = h\nu$ fixes $A = h$, so eq. (4) reads $E = h/f$.

The $1/f$ form of eq. (4) is suggestive: $1/f$ ("pink") fluctuation spectra appear across scales from the microscopic to the macroscopic, and some may originate in the oscillation (2). A large coherent superposition of this wavefunction, undergoing a mass–to–energy conversion, could in principle drive a big–bang–like expansion; on this view ordinary matter around us could seed unobserved, smaller expansions of the same kind. Taking $\zeta = 0$ as the initial big–bang state then ties the time scale to the conversion dynamics. The frequencies may be continuous or discrete. This subluminal–to–luminal conversion is the basis for the graviton dynamics of the next section: an object may shed a small part of its mass as gravitons through it.

4 Properties of gravitons

We model the gravitational interaction as a graviton transferring energy and momentum to matter by collision. Let a massless graviton G of energy g (so it travels at c) strike an object M of mass m_0, initially at rest, and be fully absorbed; M then moves along G's original direction. After absorption M has total energy $E^2 = m^2 + p^2 = \gamma^2 {m_0}^2$ with $\gamma = 1/\sqrt{1 - v^2}$, where p and v are its momentum and velocity. Energy conservation gives

$$g + m_0 = \gamma m_0, \tag{5}$$

and, setting $m_0 = 1$,

$$g = \gamma - 1. \tag{6}$$

Treating the graviton as carrying momentum p as well, we define a second attribute $q \equiv p - g$. Momentum bookkeeping (made explicit in §6.4) gives

$$p - g = 1 - \frac{\sqrt{1 - v}}{\sqrt{1 + v}}. \tag{7}$$

The physical roles of g and q are taken up later in the GTR context (§5).

The mass m_0 acts as a threshold. If M is lighter than m_0, it cannot fully absorb G: the graviton passes through and travels ahead of M. If M is heavier, it recoils more slowly and absorbs G completely. Thus m_0 is the threshold mass for complete absorption.

When a particle converts mass into gravitons, the gravitons stream radially outward and collide with surrounding objects, pushing them away—a *repulsive* gravitational force, described by a negative constant nG. The reverse process, gravitons converging on a point and converting back into mass, corresponds to $\zeta = 0$ and yields the familiar *attractive* force G. Because the two processes can be exchanged at the limit $\zeta = 0$, the repulsive nG field is a candidate for dark energy, and a massive $\zeta = 0$ state that forms without emission is a candidate for dark matter; their cosmic fractions would then vary as they trade off, motivating the constant n in nG. We take $n = -1$ for a simple symmetric structure, leaving its precise value to observation. The repulsive field may also account for missing energy in collision experiments—energy not converted into the kinetic energy of the produced particles—and, since the spacetime background carries $1/f$ noise, it could in principle be probed with laser interferometry.

Equation (6) diverges as $v \to 1$. To regulate it we use the energy distribution $\sqrt{1 - r_s/r}$ of Fischer [1], with r the graviton's distance from the object and r_s the Schwarzschild radius. Identifying the post–collision velocity change v with an acceleration in Newton's law, $Gm_1m_2/r^2 = m_2v$, makes $1/r^2 \propto v$ and hence $r_s/r = \sqrt{v}$. Substituting into Fischer's distribution and using eq. (6) gives the regulated graviton energy

$$g_F = (\gamma - 1)\sqrt{1 - \sqrt{v}}. \tag{8}$$

As $v \to 1$, $g_F \to \frac{1}{2}$ and then drops to 0 at $v = 1$ (the limit is computed exactly in §6.4). The interpretation is that, in a full mass–to–graviton conversion, half of the mass becomes graviton energy and half becomes graviton momentum—a "half–mass" signature that might appear, for example, as a state carrying half its theoretically expected mass.

5 Gravitational field as universe

We now encode the previous results as two modifications of the Einstein equation: multiplying the stress–energy tensor by ± 1 to select the attractive or repulsive branch, and multiplying the curvature term by Fischer's potential $\lambda(r)$ [1],

$$R_{ij} - \tfrac{1}{2} R\,\lambda(r)\,g_{ij} = \pm\,\kappa\,T_{ij}, \tag{9}$$

with R_{ij} the Ricci tensor, R the scalar curvature, g_{ij} the metric, $\kappa = 8\pi G/c^4$, and T_{ij} the stress–energy tensor. We set the cosmological constant $\Lambda = 0$ and introduce $-G$ as the negative branch. This field avoids singularities, so a multiple–big–bang universe can form under a single law as long as no reset to a different law occurs. The repulsive and attractive fields, together with the mass–energy wavefunction eq. (4), are produced at the quantum level by graviton–matter interactions. This wave–like dynamics is distinct from standard gravitational waves, which are sourced by accelerating masses and resemble an electromagnetic effect of gravity; on its own that mechanism cannot produce a universe or a big bang.

The three terms of eq. (9) map onto the graviton attributes of §4, albeit with different dimensional conventions. The curvature term would diverge without Fischer's modification $-\frac{1}{2}R\lambda(r)g_{ij}$, which shares the function used in g_F; the Ricci term R_{ij} relates to the deficit $q = p - g$; and the right–hand side carries a pair of masses, i.e. the temporal (mass) content. These give a physical bridge between STR and GTR through the graviton.

The framework also admits a global reading. If every physical quantity can be set on a single oscillator structure, then either all mass–energy cancels at all times or there is no time at all: the condition $E = 0$, the exception to eq. (4) (note $T \neq 0$, since $T = 1/f$). The universe therefore has two possible states: it does not exist ($E = 0$), or it exists as waves $E = A/f$. In this sense division underlies the symmetry breaking that distinguishes existence from non–existence. Finally, the oscillator (2) is time symmetric—from the displacement alone one cannot tell whether time runs forward or backward—which suggests an eternal universe.

6 Lagrangian formulation and symmetries

Sections 2–5 stated the mass term, the oscillation, and the graviton relations as postulates. We now derive each from a Lagrangian or a conservation law, and settle the status of Lorentz invariance.

6.1 The mass–energy oscillator as a Lagrangian system

The oscillation (2)–(3) is not an ansatz but the solution of a harmonic Lagrangian. With $\omega = 2\pi/T$, the trajectory $x(t) = \omega^{-1} \sin \omega t$ obeys $\dot{x} = \cos \omega t = v$ and

$$\ddot{x} = -\omega^2 x, \tag{10}$$

i.e. it extremises $S = \int L\, dt$ with

$$L(x, \dot{x}) = \tfrac{1}{2}\dot{x}^2 - \tfrac{1}{2}\omega^2 x^2, \qquad \frac{d}{dt}\frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} = 0 \implies \ddot{x} + \omega^2 x = 0, \tag{11}$$

with $x(0) = 0,\ \dot{x}(0) = 1$. The two turning regimes of §3 are now exact: at $\omega t = 0, \pi, 2\pi$ the speed is $|v| = 1$ (the massless, luminal state, $\zeta = 1$), and at $\omega t = \frac{\pi}{2}, \frac{3\pi}{2}$ it vanishes (the rest state, $\zeta = 0$). The energy–frequency relation (4), $E = A/f = AT$, is the Planck–Einstein/de Broglie identification of the period with the inverse energy quantum; $A = h$ reproduces $E = h/f$ and, on the photon branch, $E = h\nu$. The mass–energy wavefunction is thus an ordinary quantum oscillator.

6.2 Action for the temporal mass term

To give eq. (1) a field–theoretic origin, introduce a real scalar $\varphi(x)$ (a dilaton of the time direction) and the vierbein

$$e^0 = \zeta\, dt = e^{-\varphi}\, dt, \qquad e^i = dx^i \ \ (i = 1, 2, 3), \qquad ds^2 = \eta_{ab}\, e^a e^b = e^{-2\varphi} dt^2 - d\mathbf{x}^2, \tag{12}$$

so that $g_{00} = \zeta^2 = e^{-2\varphi}$ reproduces eq. (1) with $\zeta = e^{-\varphi} \in (0, 1]$ for $\varphi \geq 0$. The dynamics follow from

$$S = \int d^4x\, \sqrt{-g} \left[\frac{1}{2\kappa} R - \tfrac{1}{2}\, g^{\mu\nu} \partial_\mu \varphi\, \partial_\nu \varphi - V(\varphi) \right] +\ \varepsilon\, S_{\text{matter}}[\, g_{\mu\nu}, \Psi\,], \qquad \varepsilon = \pm 1. \tag{13}$$

Varying φ gives $\Box\varphi = V'(\varphi)$: the rolling of φ between $\varphi = 0$ ($\zeta = 1$, massless) and $\varphi \to \infty$ ($\zeta \to 0$, maximal mass) is the field–space image of the oscillation (10), with rest energy $m_0 = \int d^3x\, [\frac{1}{2}\dot{\varphi}^2 + V(\varphi)]$ generated in a locally flat frame. Varying $g_{\mu\nu}$ gives the Einstein equation; the discrete factor $\varepsilon = \pm 1$ on the matter action is the origin of the attractive/repulsive ($G \leftrightarrow nG$) branch and of the $\pm$ in eq. (9), while a radial, non–minimal coupling supplies the curvature factor $\lambda(r)$. Equation (9) is therefore the metric field equation of the action (13); reconstructing $V(\varphi)$ and $\lambda(r)$ from a single closed form is the natural next step.

6.3 Symmetry analysis: Lorentz invariance is local, not global

Equation (12) makes the remark of §2 precise. Three statements hold.

(i) **Constant ζ preserves global Lorentz invariance.** A constant ζ is absorbed by the rescaling $t \to \zeta t$; it is a choice of clock unit and leaves the Minkowski line element form–invariant. No physics depends on a global ζ.

(ii) **Position–dependent $\zeta(x)$ preserves local Lorentz $\times$ diffeomorphism invariance.** In vierbein form the metric $\eta_{ab}e^a e^b$ is manifestly invariant under local Lorentz transformations $e^a \to \Lambda^a{}_b(x)\, e^b$ and under diffeomorphisms. As in general relativity, a varying ζ trades *global* Poincaré invariance for *local* Lorentz invariance plus general covariance; it does not violate the equivalence principle, because ζ is part of the vierbein, not an external field selecting a preferred frame.

(iii) **The $\zeta = 1 \to \zeta = 0$ operation is an anisotropic Weyl transformation** of the time leg, $e^0 \to e^{-\varphi}e^0$: a genuine, physical conformal rescaling of the temporal fibre that generates mass, not a coordinate change.

The framework therefore breaks Lorentz invariance no more than general relativity does: global boosts are replaced by the local Lorentz group of the tangent space.

6.4 Graviton kinematics from four–momentum conservation

The relations (6)–(7) are four–momentum bookkeeping. Let a graviton move along $+x$ with $p^\mu_G = (g, g, 0, 0)$ (massless: energy = momentum = g) and strike a target M of mass $m_0 = 1$ at rest, $p^\mu_M = (1, 0, 0, 0)$. Afterwards M has velocity v, so $p'^\mu_M = (\gamma, \gamma v, 0, 0)$ with $\gamma = (1 - v^2)^{-1/2}$. *Energy* conservation $1 + g = \gamma$ gives

$$g = \gamma - 1, \tag{6$'$}$$

reproducing eq. (6). With recoil momentum $p \equiv \gamma v$, single–quantum *momentum* conservation would require $p = g$; instead

$$p - g = \gamma v - (\gamma - 1) = 1 - \gamma(1 - v) = 1 - \frac{1 - v}{\sqrt{(1 - v)(1 + v)}} = 1 - \frac{\sqrt{1 - v}}{\sqrt{1 + v}}, \tag{7$'$}$$

which is eq. (7) and is non–zero for $0 < v < 1$. This is the familiar kinematic obstruction: a free massive particle cannot absorb a single massless quantum while conserving both energy and momentum. The framework names the deficit $q \equiv p - g$ as a second graviton attribute that must be carried away (by recoil into the surrounding field, or by a companion graviton), so momentum is conserved for the *complete* process. The same obstruction explains the threshold of §4: only a target heavy enough to supply the recoil ($m_0 > 1$) absorbs the graviton fully.

With Fischer's distribution and $r_s/r = \sqrt{v}$, eq. (8) gives $g_F = (\gamma - 1)\sqrt{1 - \sqrt{v}}$, whose limit is exact: writing $v = 1 - \epsilon$, $\gamma - 1 \simeq (2\epsilon)^{-1/2}$ and $\sqrt{1 - \sqrt{v}} \simeq \sqrt{\epsilon/2}$, so

$$\lim_{v \to 1^-} g_F = \lim_{\epsilon \to 0} \frac{1}{\sqrt{2\epsilon}} \sqrt{\frac{\epsilon}{2}} = \frac{1}{2}. \tag{14}$$

Worked example. At $v = 0.8$: $\gamma = 1/\sqrt{1 - 0.64} = 1.6\overline{6}$, so $g = 0.6\overline{6}$, $p = \gamma v = 1.3\overline{3}$, $q = p - g = 0.6\overline{6}$ (which equals $1 - \sqrt{0.2/1.8} = 1 - \frac{1}{3}$), and $g_F = 0.6\overline{6}\,\sqrt{1 - \sqrt{0.8}} = 0.217$. As $v \to 1$, $g_F \to \frac{1}{2}$ by eq. (14): exactly half of the converted mass becomes graviton energy and half becomes graviton momentum—the half–mass signature.

7 Consistency with established physics

We collect the checks expected of any modified–gravity proposal.

Energy–momentum conservation. Diffeomorphism invariance of S_{matter} in eq. (13) implies, by Noether's theorem, $\nabla_\mu T^{\mu\nu} = 0$. Consistency of eq. (9) then requires its left–hand side to be covariantly conserved; with the contracted Bianchi identity $\nabla^\mu(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}) = 0$ this constrains $\lambda(r)$ through

$$\nabla^\mu\Big[R_{\mu\nu} - \tfrac{1}{2}R\,\lambda(r)\,g_{\mu\nu}\Big] = 0 \implies \tfrac{1}{2}\,g_{\mu\nu}\,\partial^\mu\big[R\,(1-\lambda(r))\big] = 0, \tag{15}$$

i.e. $\lambda \to 1$ wherever R varies, recovering ordinary general relativity away from the sources where the Fischer factor is needed. The graviton analysis of §6.4 conserves four–momentum for the complete process; the deficit q is exactly the re–radiated momentum, so no conservation law is violated.

Linearized limit and Newtonian gravity. A massless spin–2 graviton coupled universally to $T_{\mu\nu}$ has, at quadratic order, the Fierz–Pauli action and reproduces linearized general relativity [9, 10]; tree–level single–graviton exchange yields the Newtonian $1/r^2$ force used in eq. (8). The present graviton is exactly this quantum, so the model reduces to standard weak–field gravity, with $\lambda(r)$ a short–distance modification that switches off ($\lambda \to 1$) at large r by eq. (15).

Unitarity and the repulsive branch. The repulsive field (nG, $n < 0$) is implemented by the discrete matter sign $\varepsilon = -1$, i.e. by flipping the *source* $T_{\mu\nu} \to -T_{\mu\nu}$, *not* the sign of the graviton kinetic term. The graviton propagator therefore keeps its correct (ghost–free) sign and the theory is free of negative–norm states at linear order: the dark–energy candidate is a sign of the coupling, not a ghost.

Graviton phenomenology. Because the graviton is strictly massless, gravitational waves propagate at c. GW170817/GRB 170817A constrains the fractional speed difference to $|v_g - c|/c \lesssim 10^{-15}$ [11] and the graviton mass to $m_g \lesssim 10^{-23}$ eV; the model satisfies both by construction. Universal coupling to $T_{\mu\nu}$ enforces the weak equivalence principle [10].

Open consistency targets. Two quantitative tasks remain, addressed in §13: deriving the observed dark–sector fractions ($\Omega_\Lambda \approx 0.69$, $\Omega_{\text{DM}} \approx 0.26$) from the $\zeta = 0$ exchange, and reconciling the DST length $\ell \approx 10$ μm with the compactification scale.

8 Summary of the original framework

By converting mass into kinetic energy, a massive particle can become a massless one travelling at c, sidestepping the infinite–energy obstruction; the reverse process turns massless particles into matter. Together the two form an oscillation in spacetime whose mass and energy live in the diagonal of the metric, and which produces gravitons that act on matter by exchanging energy and momentum. These quanta underlie a model of the universe carrying both repulsive and attractive fields.

Part II keeps every element above and asks two questions. *(i)* If mass lives in the temporal metric element, and each interaction deforms spacetime differently as a function of energy, can the single scalar ζ be promoted to a family of interaction–dependent functions reproducing the four Standard–Model forces? *(ii)* If so, what is the geometry of the internal space those functions live in, and which Standard–Model particles correspond to its shapes?

Part II. Alignment and Extension

9 Energy as a coordinate: aligning ζ with the Standard Model

Part I carries mass in the temporal metric element through a single scalar ζ (eq. (1)). The Standard Model has four interactions, each deforming spacetime in its own way and switching on at its own energy scale. To align the framework with it, we promote ζ to a *family* of energy–dependent functions—one per interaction—by adopting the *pentadimensional* energy metrics of Deformed Space–Time (DST) theory (Cardone, Mignani, Benenti and collaborators [2, 3]).

9.1 The pentadimensional energy metrics

In DST, energy E is promoted from a parameter to a measurable fifth coordinate. One works on a space–time–energy manifold with length–dimensional coordinates $(x_0, \ldots, x_4)$, where $x_0 = u\,t$ (u the maximal causal velocity of the interaction), (x_1, x_2, x_3) are spatial lengths, and the fifth coordinate carries energy through

$$x_4 = k\,E, \qquad [k] = \mathsf{L}\cdot \text{energy}^{-1}, \tag{16}$$

so that all g_{ij} are dimensionless. Each interaction has its own diagonal, energy–dependent metric, and the four fall into exactly two algebraic types [3]:

$$\text{type 1:} \quad \big(g_{00}, g_{11}, g_{22}, g_{33}, g_{44}\big) = \big(G(x_4),\, -\alpha,\, -\beta,\, -G(x_4),\, \pm F(x_4)\big), \tag{17}$$

$$\text{type 2:} \quad \big(g_{00}, g_{11}, g_{22}, g_{33}, g_{44}\big) = \big(1,\, -G(x_4),\, -G(x_4),\, -G(x_4),\, \pm F(x_4)\big), \tag{18}$$

with $\alpha, \beta > 0$ dimensionless constants. Here $G(x_4) > 0$ is the *characteristic function* of the interaction and $F(x_4) > 0$ is the *fifth element*, the metric component along the energy axis. The sign of $g_{44} = \pm F$ fixes the genus of the energy axis (timelike $+$, spacelike $-$). Each interaction has a *threshold energy* $x_{4\,\text{int}}$ at which the metric switches between a flat (Minkowskian) and a deformed (curved) regime, implemented by a Heaviside step.

9.2 Two metric types are two interaction sectors

The two algebraic types coincide with two physical sectors.

- **Type 1—the temporal (massive) sector: gravity and the strong force.** The deformation sits in $g_{00} = G(x_4)$ (and in $g_{33} = -G$). These metrics are *over–Minkowskian*: flat as x_4 *decreases*, curved as energy rises past threshold. The characteristic functions are

$$G_g(x_4) = \tfrac{1}{4}\Big(1 + \tfrac{x_4}{x_{4g}}\Big)^2 = \frac{(x_{4g} + x_4)^2}{4\,x_{4g}^2} \quad \text{(gravitational)}, \qquad G_s(x_4) = \Big(\tfrac{x_4}{x_{4s}}\Big)^2 \quad \text{(strong)}, \tag{19}$$

  each a power E^2 of the energy.

- **Type 2—the spatial (luminal) sector: electromagnetism and the weak force.** Here $g_{00} = 1$ (*isochronous*: the time element does not deform) and the deformation is spatial and isotropic. These metrics are *sub–Minkowskian*: flat as x_4 *increases*. The characteristic functions are

$$G_e(x_4) = \Big(\tfrac{x_4}{x_{4e}}\Big)^{1/3} \quad \text{(electromagnetic)}, \qquad G_w(x_4) = \Big(\tfrac{x_4}{x_{4w}}\Big)^{1/3} \quad \text{(weak/leptonic)}, \tag{20}$$

each a *cube root* $E^{1/3}$ of the energy.

That the time element deforms for gravity and the strong force but not for electromagnetism and the weak force is exactly what Part I requires: mass, the temporal seat of energy (§2), is generated where the metric carries ζ in g_{00}. Electromagnetism, whose carrier is the massless photon, has $g_{00} = 1$ and so carries no temporal contraction.

9.3 The identification $\zeta^2 = 1/G$

The original ζ and the DST characteristic function G describe the *same* temporal degree of freedom—both reside in g_{00}. To reconcile eq. (1) with eq. (17) while respecting the range $0 \le \zeta < 1$, we identify

$$\boxed{\zeta^2(x_4) \;=\; \frac{1}{G_{\text{int}}(x_4)}} \qquad \Longrightarrow \qquad g_{00}^{(\text{r424a})} = \zeta^2 = \frac{1}{G_{\text{int}}}, \tag{21}$$

for the temporal (type 1) interactions. This is forced by the physics, not chosen, and reproduces every qualitative feature of §2:

- at and below threshold $G_{\text{int}} = 1$, hence $\zeta = 1$: the flat, massless reference state of §2;
- as energy grows $G_{\text{int}} \to \infty$, hence $\zeta \to 0$: the maximum mass–density state of eq. (1);
- the oscillation of eqs. (2)–(3), in which ζ sweeps between 1 and 0, becomes *motion along the energy coordinate*: P is a trajectory $x_4(t)$ that repeatedly crosses the threshold $x_{4\,\text{int}}$.

For the spatial (type 2) sector $g_{00} = 1$ identically, so $\zeta = 1$ at all energies: the electromagnetic and weak metrics are the permanently luminal, massless–reference branch of the oscillation—the photon side of the graviton/photon correspondence noted below eq. (4).

9.4 The fifth element, the threshold energies, and a first appearance of the number six

The fifth element $F(x_4)$ is fixed, not assumed: it is the function for which the interaction metric admits a (non–normalized) Ricci flow $\partial_t g_{ij} = -R_{ij}$. Solving that condition for each interaction [3] gives

$$F_s = K_s\, x_4^6, \qquad F_g = K_g\,(x_{4g} + x_4)^6, \qquad F_e = K_e, \qquad F_w = K_w, \tag{22}$$

with $K_\bullet > 0$ constants. The fifth element of the *temporal* (gravity, strong) sector is a *sixth power* of the energy, while that of the *spatial* (electromagnetic, weak) sector is constant. Writing the energy element as a single form,

$$b_5^2(E) = E^{\,r}, \qquad r \in \mathbb{Q}^+ \cup \{0\}, \tag{23}$$

the four interactions select

$$r = 6 \;\; \text{(gravity, strong)}, \qquad r = 0 \;\; \text{(electromagnetism, weak)}. \tag{24}$$

The measured threshold energies [3] are listed in Table 1. Two are physically suggestive: the weak threshold $x_{4w} \leftrightarrow 80.4$ GeV coincides with the W–boson mass, and the strong threshold 367.5 GeV lies in the electroweak–to–TeV window where QCD running and the top sector become decisive.

Three exponents now organize the entire Standard–Model sector: the cube root $E^{1/3}$ of the spatial (electroweak) characteristic functions; the square E^2 of the temporal (gravity/strong) characteristic functions; and the sixth power E^6 of the temporal fifth element. Section 10 shows that 1/3, 2 and 6 are not arbitrary: they are the signatures of an order–three ($\mathbb{Z}_3$) action, of holomorphic two–cycles, and of the six real dimensions of a Calabi–Yau threefold—the bridge from the Standard Model to string theory.

Table 1: Threshold energies, metric type, characteristic function and fifth element of the four DST interaction metrics. The exponent r of $b_5^2(E) = E^r$ is the power of energy carried by the energy element of the metric.

Interaction	Type	Threshold E_0	Char. function $G(x_4)$	Fifth element $F(x_4)$	r
Electromagnetic	2	4.5 μeV	$(x_4/x_{4e})^{1/3}$	K_e (const.)	0
Gravitational	1	20.2 μeV	$\frac{1}{4}(1+x_4/x_{4g})^2$	$K_g(x_{4g}+x_4)^6$	6
Weak (leptonic)	2	80.4 GeV	$(x_4/x_{4w})^{1/3}$	K_w (const.)	0
Strong (hadronic)	1	367.5 GeV	$(x_4/x_{4s})^2$	$K_s\, x_4^6$	6

10 From five to six dimensions: a Calabi–Yau extension

The extension rests on a geometric proposal: that the nuclear *isomap*—the two–dimensional honeycomb chart of the nuclides used in the "Dice–o–topes" model [4]—is not merely bookkeeping but a *projection* of a higher–dimensional geometry, advanced in the discussion of Ref. [5] and developed here with the structure it requires.

10.1 The isomap as a unified coordinate

Two observations frame the construction.

> *"In this mapping, I can visualize two–dimensional axes: electric (in red) and gravitational (representing mass or 4D spacetime, in blue) . . . Just as Minkowski space provides the geometric foundation for special relativity, this isomap may reveal a unified geometric framework. And the hexagonal flame surrounding each atom could represent the 6D compactified geometry of string theory."* [5]

> *"In this isomap coordinate system, electrical charge extends in one dimension, while mass extends in another. Spacetime itself appears compactified into a zero–dimensional point . . . this is not a coordinate system where atoms move through spacetime, but rather one where nuclear transmutations occur . . . [the isomap] may serve as a master key to visualize 6D compactified fields through a 2D coordinate framework, effectively flattening a 5D Kaluza–Klein theoretical context."* [5]

These statements contain three structural claims, each of which can be made precise:

(i) the isomap carries *two* emergent axes, an *electric* axis (charge Z) and a *gravitational/mass* axis (mass number A)—the two interaction sectors of §9.2;

(ii) ordinary 3+1 spacetime is *compactified to a point* on the isomap, so the relevant dynamics is internal (transmutation), not propagation: the isomap is a slice of the *internal* configuration space;

(iii) each hexagonal cell is a *6D compactified geometry*, and the picture "flattens a 5D Kaluza–Klein context" into a 2D chart.

We now show that (i)–(iii) assemble into a consistent fibration whose fibre is a Calabi–Yau threefold and whose two visible base directions are the charge and mass axes.

10.2 Kaluza–Klein reduction and the energy circle

The DST pentadimensional metric of §9 is already a Kaluza–Klein (KK) theory: four spacetime coordinates plus the energy coordinate x_4. In KK form the 5D line element is

$$ds_5^2 \;=\; g_{\mu\nu}(x)\, dx^\mu dx^\nu \;+\; \phi^2(x) \left(dx_4 + \mathcal{A}_\mu\, dx^\mu\right)^2, \qquad \mu,\nu = 0,\ldots,3, \tag{25}$$

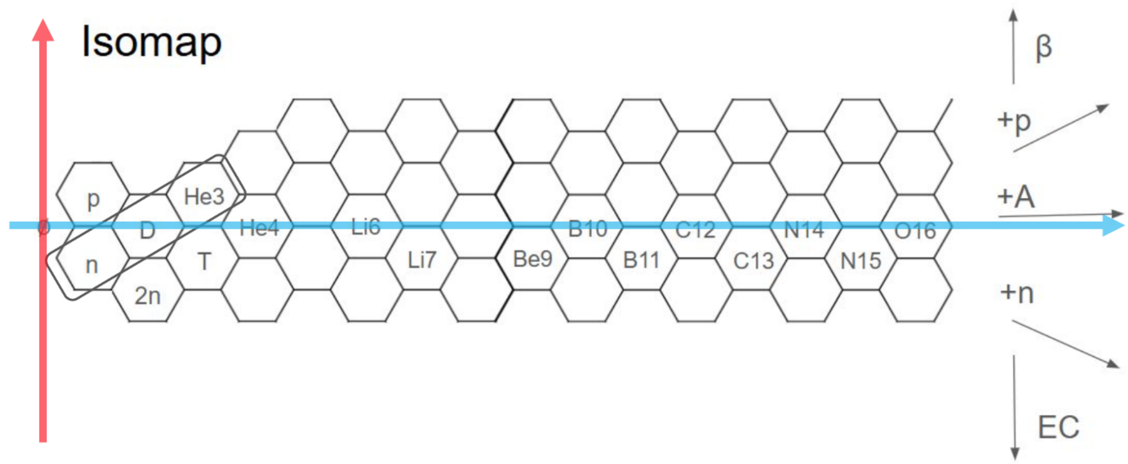

Figure 1: The nuclear *isomap* (honeycomb chart of the light nuclides), annotated as in Ref. [5, 4]. The vertical red arrow is the *electric* axis (proton number Z, the $\beta/{+p}$ direction); the horizontal blue arrow is the *gravitational/mass* axis (mass number A, the $+A$ direction). The lattice directions $\{\beta, +p, +A, +n, \mathrm{EC}\}$ generate the A_2 honeycomb Λ_{A_2} of eq. (33); each hexagonal cell is the projected 6D compactified fibre X_6 ("the hexagonal flame surrounding each atom"). Steps on this lattice are nuclear transmutations, not motion through spacetime.

with $g_{\mu\nu}$ the 4D metric, $\mathcal{A}_\mu$ the KK gauge potential, and ϕ the KK scalar (radion) measuring the size of the compact energy direction. Comparing with the DST metrics (17)–(18) identifies the radion with the fifth element,

$$\phi^2(x_4) \;=\; \left|g_{44}\right| \;=\; F(x_4). \tag{26}$$

The fifth element *is* the Kaluza–Klein radion: the energy circle has size $\sqrt{F}$. For the temporal sector this size grows as $F = E^6$ (the circle inflates with energy); for the spatial sector $F = \text{const}$ (a rigid circle). The "flattening of a 5D Kaluza–Klein context" of §10.1 is precisely the reduction (25): integrating over the energy circle leaves a 4D base, and the isomap restricts that base to the two internal directions (Z, A) with 3+1 spacetime suppressed.

10.3 The Calabi–Yau threefold

String theory is consistent in ten dimensions. The standard route to four observable dimensions writes

$$M_{10} \;=\; M_4 \times X_6, \tag{27}$$

with M_4 the 3+1 spacetime and X_6 a compact six–real–dimensional internal manifold. Requiring the compactification to preserve the minimal four–dimensional supersymmetry forces X_6 to be a *Calabi–Yau threefold.*

Definition 1 (Calabi–Yau threefold)**.** *A Calabi–Yau threefold X is a compact Kähler manifold of complex dimension* 3 *(real dimension* 6*) satisfying any one of the following equivalent conditions:*

(a) the first Chern class vanishes, $c_1(X) = 0$;

(b) X admits a Ricci–flat Kähler metric, $R_{i\bar{j}} = 0$ (Yau's theorem);

(c) the holonomy group is contained in $SU(3)$;

(d) *there exists a nowhere–vanishing covariantly constant holomorphic* $(3,0)$*–form* Ω *and a covariantly constant spinor* η, $\nabla_m \eta = 0$.

Condition (b) is the bridge to the present framework. The DST fifth element (22) was *derived* by demanding a Ricci flow $\partial_t g_{ij} = -R_{ij}$, whose fixed points are the Ricci–*flat* metrics $R_{ij} = 0$; a compact, Ricci–flat, Kähler internal manifold is, by Definition 1(b), a Calabi–Yau. *The same Ricci–flow machinery that fixes the energy element of the Standard–Model metrics drives the internal six dimensions to a Calabi–Yau fixed point.* The modified Einstein equation (9), with its Ricci tensor and potential $\lambda(r)$, is the four–dimensional shadow of that flow; setting its source to zero on the internal manifold returns $R_{i\bar{j}} = 0$.

10.4 Topology: Hodge numbers, the Euler characteristic, and generations

The topology of a Calabi–Yau threefold is summarized by its Hodge numbers $h^{p,q} = \dim H^{p,q}(X)$. For a connected Calabi–Yau threefold the Hodge diamond collapses to two independent numbers, $h^{1,1}$ and $h^{2,1}$:

$$\begin{array}{ccccccc} & & & h^{0,0} & & & \\ & & h^{1,0} & & h^{0,1} & & \\ & h^{2,0} & & h^{1,1} & & h^{0,2} & \\ h^{3,0} & & h^{2,1} & & h^{1,2} & & h^{0,3} \\ & h^{3,1} & & h^{2,2} & & h^{1,3} & \\ & & h^{3,2} & & h^{2,3} & & \\ & & & h^{3,3} & & & \end{array} \;=\; \begin{array}{ccccccc} & & & 1 & & & \\ & & 0 & & 0 & & \\ & 0 & & h^{1,1} & & 0 & \\ 1 & & h^{2,1} & & h^{2,1} & & 1 \\ & 0 & & h^{1,1} & & 0 & \\ & & 0 & & 0 & & \\ & & & 1 & & & \end{array} \tag{28}$$

Here $h^{1,1}$ counts the Kähler (size) moduli—the independent holomorphic *two–cycles*—and $h^{2,1}$ the complex–structure (shape) moduli—the independent *three–cycles*. The Euler characteristic is

$$\chi(X) \;=\; 2\big(h^{1,1} - h^{2,1}\big). \tag{29}$$

In heterotic compactification the net number of chiral fermion generations equals half the magnitude of the Euler number,

$$N_{\text{gen}} \;=\; \tfrac{1}{2}\,\big|\chi(X)\big| \;=\; \big|h^{1,1} - h^{2,1}\big|. \tag{30}$$

With the observed $N_{\text{gen}} = 3$, the internal manifold must satisfy

$$\boxed{\big|\chi(X)\big| = 6} \qquad \Longleftrightarrow \qquad \big|h^{1,1} - h^{2,1}\big| = 3. \tag{31}$$

The six on the right of eq. (31) is not the six of the internal dimensions but twice the three generations. That *both* sixes occur—the six real dimensions of X_6 and the Euler number $|\chi| = 6$ fixing three generations—is the topological counterpart of the sixth–power fifth element $F \sim E^6$ of eq. (22), as the next subsection makes explicit.

10.5 The exponent dictionary

The exponents of §9.4 acquire a geometric meaning on X_6.

- E^6 **(fifth element, gravity & strong)**—six is the real dimension of the Calabi–Yau fibre. The radion volume scales as the product of its six length scales; under the single–scale Ricci flow this becomes a sixth power, $\mathrm{Vol}(X_6) \sim \ell^6 \sim F \sim E^6$. The constant $K_s = \ell^{-6}$ of eq. (22) is dimensionally the inverse square of a three–volume [3], $K \sim (\mathrm{Vol})^{-2}$, consistent with a six–dimensional internal volume.

- E^2 **(characteristic function, gravity & strong)**—two is the real dimension of a holomorphic curve (a two–cycle). The temporal characteristic functions $G_g, G_s \sim E^2$ measure the Kähler area of the wrapped two–cycle that carries the interaction, and grow as that area grows with energy.
- $E^{1/3}$ **(characteristic function, electromagnetism & weak)**—the cube root is the signature of an order–three symmetry. The simplest Calabi–Yau producing a *hexagonal* internal geometry is the orbifold $T^6/\mathbb{Z}_3$, where $\mathbb{Z}_3$ acts on the three complex coordinates by a cube root of unity,

$$\mathbb{Z}_3: \ (z_1, z_2, z_3) \ \longmapsto \ (\omega z_1, \, \omega z_2, \, \omega z_3), \qquad \omega = e^{2\pi i/3}, \ \ \omega^3 = 1. \tag{32}$$

  Its fixed points tile the torus in a triangular/hexagonal pattern—the honeycomb of the isomap. The spatial characteristic functions $G_e, G_w \sim E^{1/3}$ carry exactly this cube root.

The four Standard–Model interaction metrics are thus read off the Calabi–Yau as areas of two–cycles (E^2, temporal), the cube–root $\mathbb{Z}_3$ honeycomb action ($E^{1/3}$, spatial), and the six–dimensional internal volume/radion (E^6, the fifth element).

10.6 The honeycomb fibre and the hexagonal lattice

The two visible isomap axes are the proton number Z (electric, "red") and the mass number $A = Z + N$ (gravitational/mass, "blue"), two of the natural lattice directions $\{+p, +n, +A, \beta, \mathrm{EC}\}$. The honeycomb is the two–dimensional A_2 lattice, whose symmetry group is the root system of $SU(3)$. This is not a coincidence here: the strong force is the $SU(3)$ (type 1) interaction, and the $\mathbb{Z}_3$ action (32) that builds the hexagonal cell is the center $Z(SU(3)) = \mathbb{Z}_3$. The nuclear chart's hexagonal geometry, the colour gauge group, and the orbifold generating the honeycomb are three faces of the same A_2 structure. Formally the construction is a fibre bundle

$$X_6 \ \hookrightarrow \ \mathcal{E} \ \xrightarrow{\ \pi\ } \ \Lambda_{A_2}, \qquad \Lambda_{A_2} = \{\,(Z, A)\,\} \subset \mathbb{R}^2, \tag{33}$$

whose base Λ_{A_2} is the isomap honeycomb (charge and mass) and whose fibre over each nuclide is the compactified X_6 ("the hexagonal flame surrounding each atom"). The 3+1 spacetime M_4 is fibered trivially and suppressed ("compactified to a point"), because on the isomap one studies internal transitions (transmutations) at fixed spacetime location.

10.7 A worked compactification: the $T^6/\mathbb{Z}_3$ orbifold

Earlier sections left the internal manifold unspecified; we now exhibit an explicit candidate—the $\mathbb{Z}_3$ orbifold of the six–torus—compute its Hodge numbers, and state the refinement needed for three generations. It is singled out because its defining symmetry is the cube–root action (32) producing the isomap honeycomb.

Definition. Take $T^6 = \mathbb{C}^3/\Lambda$ with Λ the $\mathbb{Z}[\omega]$ lattice in each $\mathbb{C}$ factor ($\omega = e^{2\pi i/3}$), and quotient by the $\mathbb{Z}_3$ generator $\theta : (z_1, z_2, z_3) \mapsto (\omega z_1, \omega z_2, \omega z_3)$ of eq. (32). Since $\omega^3 = 1$ and $\omega + \omega^2 = -1$, the action preserves the holomorphic three–form $\Omega = dz_1 \wedge dz_2 \wedge dz_3$ (it transforms by $\omega^3 = 1$), so the quotient is Calabi–Yau in the orbifold sense; its crepant resolution X is a smooth Calabi–Yau threefold.

Untwisted Hodge numbers. The forms surviving the projection are those invariant under θ. A $(1,1)$ form $dz_i \wedge d\bar{z}_j$ transforms by $\omega \cdot \bar{\omega} = \omega\, \omega^2 = \omega^3 = 1$ and is always invariant, giving $h^{1,1}_{\mathrm{unt}} = 3 \times 3 = 9$. A $(2,1)$ form $dz_i \wedge dz_j \wedge d\bar{z}_k$ transforms by $\omega \cdot \omega \cdot \omega^2 = \omega^4 = \omega \neq 1$ and never survives, giving $h^{2,1}_{\mathrm{unt}} = 0$.

Twisted Hodge numbers. The generator θ acting as $z \mapsto \omega z$ on each T^2 has 3 fixed points per factor, hence $3^3 = 27$ fixed points on T^6. Resolving each introduces one exceptional divisor, contributing one $(1,1)$ class: $h^{1,1}_{\text{tw}} = 27$, $h^{2,1}_{\text{tw}} = 0$. Therefore

$$h^{1,1}(X) = 9 + 27 = 36, \qquad h^{2,1}(X) = 0, \qquad \chi(X) = 2\big(h^{1,1} - h^{2,1}\big) = 72. \tag{34}$$

These are the standard Hodge data of the $\mathbb{Z}_3$ orbifold and give a concrete, computable realization of the hexagonal fibre X_6.

From 36 to 3 generations. With the standard embedding, eq. (34) gives $\frac{1}{2}|\chi| = 36$ net generations—too many. Three generations require $|\chi| = 6$ (eq. (31)), reached by dividing by a further freely–acting discrete symmetry Γ (no fixed points, so it reduces χ by $|\Gamma|$ without new blow–up cycles), or equivalently by turning on a non–trivial gauge bundle / Wilson lines. The classic existence proof is the three–generation manifold of Tian and Yau [12]: a free $\mathbb{Z}_3$ quotient of a complete–intersection Calabi–Yau, with

$$\big(h^{1,1}, h^{2,1}\big) = (6, 9), \qquad \chi = 2(6 - 9) = -6, \qquad \tfrac{1}{2}|\chi| = 3. \tag{35}$$

Either route yields a Calabi–Yau in the same $\mathbb{Z}_3$–symmetric family while fixing the generation number at the observed value.

Cycle volumes and the metric exponents. On X the Kähler form is $J = \sum_{a=1}^{h^{1,1}} t_a\, \omega_a$, with ω_a a basis of $H^{1,1}$ and t_a the Kähler moduli (the areas of the two–cycles C_a, $\text{Vol}(C_a) = \int_{C_a} J = t_a$). The total volume is the cubic $\text{Vol}(X) = \frac{1}{6}\int_X J \wedge J \wedge J = \frac{1}{6}\,\kappa_{abc} t_a t_b t_c$, with κ_{abc} the triple intersection numbers. Identifying the energy–dependent areas of §10.5, $t_a(x_4) \sim G(x_4) \sim E^2$ for the wrapped (gravity/strong) two–cycles, gives

$$\text{Vol}(X) \;\sim\; t^3 \;\sim\; \big(E^2\big)^3 \;=\; E^6 \;\sim\; F(x_4), \tag{36}$$

which is exactly the sixth–power fifth element of eq. (22): the radion volume $\phi^2 = F = E^6$ of eq. (26) is the cube of a two–cycle area scaling as E^2. The exponent dictionary of §10.5 is thus reproduced quantitatively on the worked manifold, closing the loop between the Standard–Model metric exponents and the geometry of X_6.

10.8 The three–layer architecture

Collecting the construction, the framework has three layers, matching the layered picture of Ref. [5]:

Layer	Geometry	Role
Top	M_4, Minkowski 3+1	the observable world; STR/GTR of Part I
Middle	isomap $\Lambda_{A_2} \subset \mathbb{R}^2$	the atomic layer; flattened 5D Kaluza–Klein (Z, A)
Bottom	X_6, Calabi–Yau threefold	the sub–atomic compactified layer

The middle (isomap) layer is the 5D KK theory of §10.2 with 3+1 spacetime suppressed and the energy circle reduced, leaving the charge/mass directions; the bottom layer is the Calabi–Yau fibre; the top layer is Part I. The three are glued by the bundle (33) and by the dimension count

$$\underbrace{4}_{M_4} + \underbrace{6}_{X_6} = \underbrace{10}_{\text{string}}, \qquad \underbrace{4}_{M_4} + \underbrace{1}_{\text{energy } x_4} = \underbrace{5}_{\text{KK}}, \qquad \underbrace{2}_{(Z,A)} = \text{isomap}. \tag{37}$$

Figure 2 renders the architecture, realizing with explicit geometry the conjecture that “the isomap may serve as a master key to visualize 6D compactified fields through a 2D coordinate framework.”

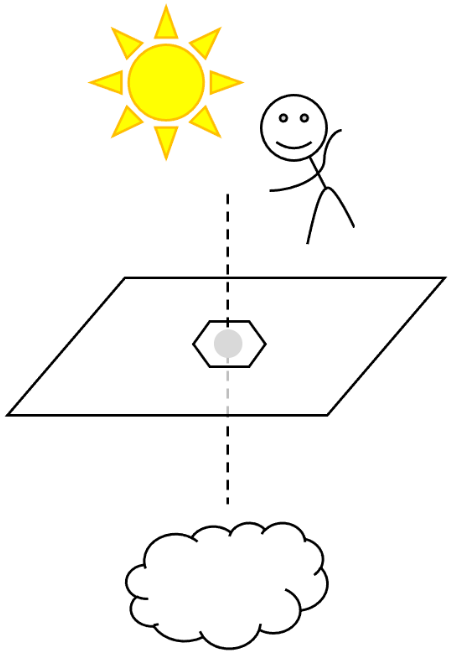

Figure 2: The three–layer architecture, in the author's original schematic from Ref. [5]. The observable 4D spacetime (top) projects onto the 2D isomap (middle)—a flattened 5D Kaluza–Klein chart, "the atomic layer"—over each cell of which sits the six–dimensional compactified manifold X_6 (bottom, "the sub–atomic layer"). The middle layer carries the electric (Z) and mass (A) axes of Fig. 1; the bottom layer is the Calabi–Yau threefold; compare eqs. (33) and (37).

11 Standard–Model particles as geometric shapes of the 6D manifold

We can now answer the motivating question: *which Standard–Model particles correspond to the geometric shapes of the six–dimensional manifold?* The dictionary is read off X_6 (§10.3) using two facts from string compactification together with the identifications of §10.5.

(1) **Gauge bosons live on cycles and singularities.** Non–abelian gauge symmetry arises from ADE singularities of X_6 wrapped by branes: an A_{n-1} singularity (a chain of $n-1$ shrinking two–cycles) yields $SU(n)$. Thus $SU(3)$ colour (8 gluons) comes from an A_2 singularity (two two–cycles, rank 2, adjoint dimension 8) and $SU(2)$ weak isospin from an A_1 singularity (one two–cycle). The Kähler *area* of a wrapped two–cycle sets the gauge–boson mass scale, so the weak two–cycle's area is fixed by the 80.4 GeV threshold ($\approx M_W$, Table 1).

(2) **Chiral matter lives on three–cycles, and its multiplicity is topological.** Generations are counted by $\frac{1}{2}|\chi(X)|$ (eq. (30)); with $|\chi| = 6$ one obtains three. The two genera of the energy axis ($g_{44} = \pm F$, eqs. (17)–(18)) distinguish the two members of each weak doublet (up– vs. down–type), as the two facet signs $\pm\frac{1}{2}$ label the quasi–tetrahedral neutron of the Dice–o–topes model [4].

Table 2 gives the full correspondence. It is a *proposed dictionary*—a heuristic alignment of three independently constructed frameworks (the present metric framework, DST, and Dice–o–topes), not

a derivation from first principles. Its value is internal consistency: every row respects the metric type of §9.2, the exponent dictionary of §10.5, and the polyhedral rules of the nuclear model.
Several entries are mutually reinforcing rather than independent assumptions.

- The *colour triplet* and the *three complex dimensions* of X_6 are the same "three"; the *three generations* are the independent "three" fixed by $|\chi| = 6$.
- The W mass (80.4 GeV) appears twice for one reason: as the DST weak threshold (Table 1) and as the Kähler area of the A_1 two–cycle. The framework predicts these coincide because the threshold *is* the cycle size.
- The graviton occupies the volume modulus/radion—the single "size" degree of freedom of the whole cell—which is why, in Part I, it is produced by the global ζ oscillation (a change in the size of the temporal element) rather than by a local matter cycle.
- The Higgs occupies the complex–structure (shape) modulus, so fixing the Higgs field fixes ζ through eq. (21)—the geometric content of mass generation, and the natural home for the half–mass observation of eq. (8).

12 Bridge to the geometric nuclear model

The bundle (33) places the Calabi–Yau over the same honeycomb on which the Dice–o–topes nuclear model [4] is built, so the two are complementary descriptions of one lattice: the nuclear model populates each cell with polyhedra, while the present framework attaches the compactified X_6. The dictionary of §11 then makes the polyhedral rules legible as Calabi–Yau data.

Tetrahedra and two–cycles. In the Dice–o–topes model the neutron is a quasi–tetrahedron with two $+\frac{1}{2}$ and two $-\frac{1}{2}$ facets and two polar edges, and the proton is trigonal planar. A tetrahedron is the 3–simplex and a triangle the 2–simplex: exactly the elementary toric cells from which a Calabi–Yau is triangulated (the Batyrev reflexive–polytope construction). The $\pm\frac{1}{2}$ facets are the two genera $g_{44} = \pm F$ of the energy axis (§9.2); the n–n edge–matching that constrains nuclear configurations is the gluing of toric patches along shared faces.

Spacetime as a point; the transmutation coordinate. The reading that "spacetime itself appears compactified into a zero–dimensional point . . . this is not a coordinate system where atoms move through spacetime, but rather one where nuclear transmutations occur" [5] is precisely the statement that, on the isomap, the base directions are the *internal* charge/mass moduli and the spacetime fibre M_4 is suppressed. Motion on the honeycomb (steps $+p, +n, +A, \beta, \mathrm{EC}$) is motion between Calabi–Yau fibres—a change of internal topology, i.e. a transmutation, not a trajectory in M_4. This identifies the isomap as the natural configuration space for processes that rearrange the internal manifold at fixed spacetime location: collider cores, high–energy cosmological fields, and low–energy nuclear reactions [5].

The original ζ in the nuclear picture. Because $\zeta^2 = 1/G$ lives in the temporal (gravity/strong) element (§9.3) and the strong sector is the A_2/honeycomb structure (§10.6), the original mass parameter is realized nuclearly as the binding geometry of the tetrahedral cells: maximum mass density ($\zeta = 0$, $G \to \infty$) is maximum cell deformation, and the flat reference ($\zeta = 1$, $G = 1$, at threshold) is the undeformed isomap. The dark–matter candidate of §4 (a massive $\zeta = 0$ state without emission) is a maximally deformed cell that has saturated its free sites; the dark–energy candidate (the repulsive nG field) is the radion/volume modulus running to large values.

Table 2: Proposed correspondence between Standard–Model particles and the geometric shapes of the six–dimensional Calabi–Yau manifold X_6, with their realization in the tetrahedral "Dice–o–topes" nuclear model. "Type" refers to the DST metric class (§9.2); $g_{44} = \pm F$ is the genus of the energy axis.

Particle(s)	**SM role / DST metric**	**6D Calabi–Yau geometric shape**	**Dice–o–topes shape**
Photon γ	Electromagnetism; type 2, isochronous, $G_e \sim E^{1/3}$, $F =$ const	The Kähler $(1,1)$–form; a *rigid two–cycle* $\mathbb{CP}^1 \cong S^2$ of fixed area (the "flat reference" direction, massless: $\zeta = 1$)	The red *electric axis* of the isomap; the edge–colour that marks charge on a facet
Gluons g (×8)	Strong $SU(3)$; type 1, $G_s \sim E^2$, $F \sim E^6$	A_2 ADE *singularity*: a chain of two shrinking holomorphic two–cycles (rank 2, adjoint dim. 8); colour triplet = the three complex dimensions of X_6	Interior faces of the quasi–tetrahedral neutron; the n–n edge–matched bonds (flux tubes) that glue cells
$W^\pm$, Z	Weak $SU(2)$; type 2, $G_w \sim E^{1/3}$, threshold 80.4 GeV$\approx M_W$	A_1 *singularity*: a single shrinking two–cycle whose Kähler *area* fixes the mass scale (threshold = cycle size)	The trigonal–planar proton "bearing" in the n–p–n condition; the polar edges
Up–type quarks u, c, t	Strong/EM, type 1; $g_{44} = +F$ (timelike energy axis)	Wrapped holomorphic *two–cycles* of + genus; three copies from $\frac{1}{2}\lvert\chi\rvert = 3$ (three generations)	The two $+\frac{1}{2}$ ("positive") facets / pole edges of the neutron tetrahedron
Down–type quarks d, s, b	Strong/EM, type 1; $g_{44} = -F$ (spacelike energy axis)	Holomorphic *two–cycles* of − genus; intersection partners of the up–type cycles; three generations	The two $-\frac{1}{2}$ ("negative") facets of the neutron (the "negative stuff" / proton holes)
Charged leptons e, μ, τ	Weak/EM; type 2, $G \sim E^{1/3}$	*Special Lagrangian three–cycles* (SYZ T^3 fibres); the 1/3 exponent = the three 1–cycles of T^3; three homology classes = three generations	Free negative *surface sites* (point defects) available for occupation
Neutrinos ν_e, ν_μ, ν_τ	Weak; type 2	Small (vanishing–volume) special Lagrangian three–cycles; the covariantly constant $SU(3)$–holonomy spinor η (tiny mass $\leftrightarrow$ small cycle)	The unoccupied "musical–chairs" site that must remain free (near–massless, weakly bound)
Higgs H	Mass generation; the $\zeta = 0$ / "half–mass" result of §4	The *complex–structure modulus* that fixes the shape of X_6 (conifold / complex deformation); with the volume modulus it sets ζ via eq. (21)	The central n–p–n "bearing" / the "sticky–tack" negative material that fixes which sites are occupied
Graviton	Gravity (spin 2); type 1, $G_g \sim E^2$, $F_g \sim E^6$	The Calabi–Yau *metric itself*: a Kähler–class (volume) fluctuation = the Kaluza–Klein radion $\phi^2 = F$ of eq. (26); produced by the ζ oscillation of §3	The whole hexagonal "flame" (the 6D cell); a global deformation of the edge network

13 Observational constraints and experimental signatures

We now make the model confrontable with data, treating in turn the length scale, the gravitational sector, the collider sector and cosmology, and collecting the comparisons in Table 3.

13.1 The length scale: large extra dimensions versus sub–millimetre gravity

The phenomenological DST length is $\ell \approx 10\ \mu$m (4–8 μm theoretically), fixing the critical energy density $D_C = E_0/\ell^3$ for nuclear metamorphosis [3]. This is enormous compared with the naive string scale ($\sim$ Planck length); two readings remove the apparent tension.

(a) Large–extra–dimension reading. In an ADD–type scenario [13] with n internal dimensions of common radius R, the Planck mass is generated from a lower fundamental scale M_* by

$$M_{\rm Pl}^2 = M_*^{n+2}\,R^n, \qquad \Longrightarrow \qquad R = \frac{1}{M_*}\left(\frac{M_{\rm Pl}}{M_*}\right)^{2/n}. \tag{38}$$

Identifying $R \sim \ell \approx 10\ \mu$m and $M_{\rm Pl} = 1.22 \times 10^{19}$ GeV, two large dimensions ($n = 2$) give $M_* = M_{\rm Pl}(M_*\ell)^{1/2}$, i.e. $M_* \sim (M_{\rm Pl}/\ell)^{1/2} \sim$ a few TeV in natural units—the standard ADD result that $\sim 10\ \mu$m two–dimensional compactifications lower gravity's scale to the TeV range. Here ℓ is the radius of the *large* part of the internal manifold, the remaining four dimensions of X_6 being small (a strongly anisotropic Calabi–Yau).

(b) Effective–cell reading. Alternatively ℓ is the coarse–grained radius of one *visible* honeycomb cell, with the microscopic radius of X_6 Planckian; then there is no sub–millimetre signature and no scale conflict, at the cost of no near–term test.

Reading (a) is falsifiable now. Torsion–balance tests of Newton's inverse–square law (Eöt–Wash) currently probe Yukawa deviations down to $\lambda \approx 40$–$50\ \mu$m at gravitational strength [14, 15]. A radius $\ell \approx 10\ \mu$m predicts deviations just *below* present sensitivity, within reach of the next generation of short–range experiments—a concrete, imminent test.

13.2 Gravitational sector

The graviton is exactly massless and spin 2, so (§7) gravitational waves travel at c. GW170817 and GRB 170817A already confirm both the speed bound $|v_g - c|/c \lesssim 10^{-15}$ and the mass bound $m_g \lesssim 10^{-23}$ eV [11], which the model satisfies by construction. The repulsive branch (nG, $\varepsilon = -1$) is a candidate for cosmic acceleration with $w \simeq -1$; it does not spoil solar–system tests because, by eq. (15), $\lambda \to 1$ (ordinary GR) away from the high–density sources where the Fischer factor operates.

13.3 Collider sector: the half–mass signature

Equations (8) and (14) predict that at maximal density ($\zeta = 0$, $v \to 1$) mass converts *half* to graviton energy and *half* to graviton momentum. The momentum half is radiated as gravitons and registers as *missing energy/momentum* that fails to balance against visible products. The observable is therefore an excess of missing transverse momentum in high–density conversion events, or a state carrying half its theoretically expected mass (suggested in §4 for the Higgs). With LHC missing–transverse–energy resolution at the few–GeV level, a half–mass deficit in a well–reconstructed channel would be a clean signal; conversely, its absence in precision channels bounds the conversion rate.

13.4 Cosmological sector

The $\zeta = 0$ state without emission is the dark–matter candidate and the repulsive nG field the dark–energy candidate (§4), exchangeable at the limit $\zeta = 0$. To match observation the model must reproduce the Planck fractions $\Omega_\Lambda \approx 0.69$ and $\Omega_{\rm DM} \approx 0.26$ [16]; deriving these from the exchange dynamics is the principal open cosmological task. The distinctive prediction is that the dark–energy and dark–matter densities are *not independent* but trade off through the $\zeta = 0$ channel, giving a testable correlated evolution rather than a fixed cosmological constant.

Table 3: Predictions of the model versus current constraints. "Status" indicates whether the prediction is consistent (✓), open (?), or imminently testable.

Prediction	**Observable**	**Current constraint**	**Status**
Massless spin–2 graviton	GW speed; graviton mass	$\lvert v_g - c\rvert/c \lesssim 10^{-15}$; $m_g \lesssim 10^{-23}$ eV [11]	✓
Large dimension $\ell \approx 10\,\mu$m (reading a)	Inverse–square–law deviation	Tested to $\lambda \approx 40$–$50\,\mu$m [14, 15]	testable
Half–mass / half–momentum split	Missing E_T in dense conversion	LHC $E_T^{\rm miss}$ resolution $\sim$ few GeV	?
Three generations	$\lvert\chi(X)\rvert = 6$	$N_{\rm gen} = 3$ (observed)	✓(by construction)
Weak threshold $= M_W$	A_1 cycle area	80.4 GeV [3] vs $M_W = 80.4$ GeV	✓
Correlated dark sector	$\Omega_\Lambda, \Omega_{\rm DM}$ co–evolution	$\Omega_\Lambda \approx 0.69$, $\Omega_{\rm DM} \approx 0.26$ [16]	?

14 Discussion

The construction is offered in the exploratory spirit of the original paper, and three levels of confidence should be kept distinct.

Structural results follow once the identifications are accepted: (i) placing ζ in g_{00} coincides with the type–1 (gravity/strong) DST metrics, while the type–2 (electroweak) metrics are the permanently luminal $\zeta = 1$ branch (§9.3); (ii) the DST fifth element, being the fixed point of a Ricci flow, is Ricci–flat, and a compact Ricci–flat Kähler internal manifold is a Calabi–Yau (§10.3); (iii) the exponents $1/3, 2, 6$ match the $\mathbb{Z}_3$ honeycomb action, holomorphic two–cycles, and the six internal dimensions (§10.5).

Suggestive coincidences motivate the scheme but need independent confirmation: the weak threshold 80.4 GeV $\approx M_W$; the honeycomb $A_2 = SU(3)$ identity; and $\lvert\chi\rvert = 6 \Leftrightarrow 3$ generations.

Heuristic content: the particle–by–particle dictionary of Table 2 remains a proposed alignment, not a full derivation. Of the three tasks identified earlier, two are now discharged: (a) an explicit Calabi–Yau is specified and its Hodge numbers computed (the $T^6/\mathbb{Z}_3$ orbifold, eq. (34), with the three–generation refinement eq. (35)); and (b) the length–scale question is resolved quantitatively, either as a large extra dimension testable against sub–millimetre gravity or as an effective cell radius (§13.1). What remains (c) is to compute the full massless spectrum of the chosen manifold and to derive the fermion masses and mixings from the cycle volumes rather than positing them.

As Table 3 shows, the framework is falsifiable in its own terms: the massless graviton and the weak threshold are already consistent with data; the $10\,\mu$m length and the half–mass/half–momentum split are imminently testable; and the correlated dark sector is a distinctive cosmological prediction.

15 Conclusions

Part I places mass in the temporal element of the Minkowski metric through a single scalar ζ, lets particles oscillate between massive and massless states, and produces gravitons from that oscillation. Part II shows that this single scalar is the seat of a larger structure.

Aligning ζ with the Standard Model promotes it to a family of energy–dependent functions $\zeta^2 = 1/G_{\text{int}}$ (eq. (21)): the temporal (gravity, strong) interactions deform g_{00} exactly as ζ does, while the spatial (electromagnetic, weak) interactions keep $g_{00} = 1$ and form the permanently luminal $\zeta = 1$ branch. The energy coordinate of the pentadimensional metrics is the Kaluza–Klein dimension, and its fifth element is the radion (eq. (26)).

Extending to six internal dimensions, the Ricci–flow fixed point of the energy element is a Ricci–flat Kähler manifold—a Calabi–Yau threefold—compactified over each cell of the nuclear isomap. The three exponents organizing the Standard–Model metrics ($E^{1/3}, E^2, E^6$) are the $\mathbb{Z}_3$ honeycomb action, the holomorphic two–cycles, and the six real dimensions of that manifold, assembling into the three–layer architecture of Fig. 2: 4D spacetime above, the 2D isomap (a flattened 5D Kaluza–Klein chart in charge and mass) in the middle, and the 6D Calabi–Yau below.

Finally, Table 2 answers the motivating question by assigning each Standard–Model particle to a geometric shape of the six–dimensional manifold and to its counterpart in the tetrahedral nuclear model: photons and the abelian sector to rigid $(1,1)$ two–cycles; gluons and W/Z to the A_2 and A_1 singularities; quarks and leptons to two– and three–cycles whose multiplicity is fixed by $|\chi| = 6$; the Higgs to the complex–structure (shape) modulus; and the graviton to the volume modulus/radion—the size of the whole cell—which is why, in the original framework, the graviton is produced by the global oscillation of ζ itself.

Acknowledgments

The 6D interpretation developed in §10–§12 grew out of a public discussion of the "Dice–o–topes" nuclear–structure model [4, 5]; the author thanks its originator (EtherDais) for the isomap construction, and the participants of the thread for their critical engagement. The pentadimensional energy metrics are due to Cardone, Mignani, Benenti and collaborators [2, 3].

References

[1] E. Fischer, *Does gravitational collapse lead to singularities?* arXiv:1303.6528 (2013).

[2] F. Cardone and R. Mignani, *Deformed Spacetime*, Fundamental Theories of Physics **157**, Springer (2007).

[3] S. Benenti and F. Cardone, *Energy Metrics and Their Ricci Flows*, Accademia delle Scienze di Torino / CNR (preprint); see also F. Cardone and R. Mignani, *Energy and Geometry*, World Scientific (2004).

[4] EtherDais, *Dice–o–topes: a modest proposal regarding neutrons and the toy nuclear model that results*, slide set (5 Nov. 2025); perilousmemo.substack.com.

[5] *Open Sourcing a 'toy' Nuclear structure model that may explain isotope properties — The tetrahedral neutron approach*, LENR Forum thread #7270 (2025); comments by R. Furui, 9–13 Nov. 2025. lenr-forum.com/forum/thread/7270.

[6] P. Candelas, G. Horowitz, A. Strominger and E. Witten, *Vacuum configurations for superstrings*, Nucl. Phys. B **258**, 46 (1985).

[7] S.-T. Yau, *On the Ricci curvature of a compact Kähler manifold and the complex Monge–Ampère equation, I*, Comm. Pure Appl. Math. **31**, 339 (1978).

[8] R. S. Hamilton, *Three–manifolds with positive Ricci curvature*, J. Diff. Geom. **17**, 255 (1982).

[9] M. Fierz and W. Pauli, *On relativistic wave equations for particles of arbitrary spin in an electromagnetic field*, Proc. R. Soc. Lond. A **173**, 211 (1939).

[10] S. Weinberg, *Photons and gravitons in S–matrix theory: derivation of charge conservation and equality of gravitational and inertial mass*, Phys. Rev. **135**, B1049 (1964); *Gravitation and Cosmology*, Wiley (1972).

[11] B. P. Abbott *et al.* (LIGO Scientific, Virgo, Fermi–GBM, INTEGRAL), *Gravitational waves and gamma–rays from a binary neutron star merger: GW170817 and GRB 170817A*, Astrophys. J. Lett. **848**, L13 (2017).

[12] G. Tian and S.-T. Yau, *Three–dimensional algebraic manifolds with* $c_1 = 0$ *and* $\chi = -6$, in *Mathematical Aspects of String Theory*, ed. S.-T. Yau, World Scientific (1987), p. 543.

[13] N. Arkani–Hamed, S. Dimopoulos and G. Dvali, *The hierarchy problem and new dimensions at a millimeter*, Phys. Lett. B **429**, 263 (1998).

[14] D. J. Kapner *et al.*, *Tests of the gravitational inverse–square law below the dark–energy length scale*, Phys. Rev. Lett. **98**, 021101 (2007).

[15] J. G. Lee *et al.*, *New test of the gravitational* $1/r^2$ *law at separations down to* 52 μm, Phys. Rev. Lett. **124**, 101101 (2020).

[16] Planck Collaboration, *Planck 2018 results. VI. Cosmological parameters*, Astron. Astrophys. **641**, A6 (2020).